UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: DGE 3113
COURSE	: DISCRETE MATHEMATICS
SEMESTER/SESSION	: 1 – 2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **7** questions. Answer **ALL** questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches whenever required.
4. If in doubt, raise your hand and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

INSTRUCTION: ANSWER ALL QUESTIONS. (100 MARKS)

QUESTION 1

a) Determine each of the following sentences is proposition or not proposition.

i) Paris is the capital of France. (1 mark)

ii) There exist integers x, y such that $x + y = 15$. (1 mark)

iii) Peel me an orange. (1 mark)

iv) It is raining today. (1 mark)

v) 2 + 5 = 10 (1 mark)

b) Let p be "The database is accessible", q be "My program can run" and r be "The output is correct".

i) Write the statement, "If the database is not accessible, then my program cannot run", into logical expressions using logical connectives. (2 marks)

ii) Express the proposition, $r \leftrightarrow (q \wedge p)$ as an English sentence. (3 marks)

c) Prove that for all integers m and n, if m is odd and n is even, then mn is even using direct proof method. (3 marks)

d) i) Construct a truth table for the compound proposition, $\sim(p \vee q) \rightarrow \sim q$.

(3 marks)

ii) Determine whether the given compound proposition as in question (d)(i) is tautology, contingency or contradiction. Justify your answer.

(2 marks)

QUESTION 2

a) List all the elements for each of the following set.

i) $P = \{x \in \mathbb{Z} \mid x$ is a prime number and less than 20$\}$ (2 marks)

ii) $Q = \{x \in \mathbb{R} \mid x^2 + 3 = 7$ and x is a positive number$\}$ (2 marks)

b) Let the universal set, $\xi = \{x \in \mathbb{R} \mid 1 \le x \le 25\}$, set $A = \{10,12,14,16,18,20,22,24\}$, $B = \{11,13,15,17,19,21,23,25\}$ and $C = \{3,6,9,12,15,18,21,24\}$. Find the elements of the following set:

i) $A \cap B$ (2 marks)

ii) $C - A'$ (2 marks)

iii) $(A \cup B)' \cap C$ (2 marks)

c) Given R_1 and R_2 be the relations on $A = \{1,2,3,4\}$ are given by

$$R_1 = \{(1,1),(1,2),(2,2),(3,4),(4,2),(4,4)\}$$

$$R_2 = \{(1,1),(1,2),(2,2),(2,3),(2,4),(3,4),(3,3)\}$$

i) Draw the digraph for each of the relations R_1 and R_1. (2 marks)

ii) Is R_1 is reflexive? Explain your answer. (2 marks)

iii) Is R_2 is transitive? Explain your answer. (2 marks)

d) Given two functions, $f(x) = 2x + 9$ and $g(x) = 1 - x^2$. Find each of the following:

i) $(f \circ f)(x)$ (2 marks)

ii) $\left(g \circ f^{-1}\right)(x)$ (3 marks)

iii) $\left(f^{-1} \circ g\right)(2)$ (3 marks)

iv) $\left(f \circ f^{-1}\right)(10)$ (3 marks)

QUESTION 3

a) Find the quotient and the remainder of the following, and state the answer in the form of **div** and **mod** respectively.

i) 459 is divided by 4 (2 marks)

ii) – 52 is divided by 3 (2 marks)

b) Convert each of the following number to its equivalent base.

i) 55 into binary number. (2 marks)

ii) 4005 into octal number. (2 marks)

iii) 1067_8 into decimal number. (2 marks)

iv) 10011110_2 into decimal number. (2 marks)

v) 1100111000010_2 into hexadecimal number. (2 marks)

QUESTION 4

a) Suppose a simple graph with five vertices of degrees 2, 2, 3, 3 and 4. How many edges does this graph have? Then, draw the graph if possible. (4 marks)

b) Draw K_5 and $K_{4,3}$. (3 marks)

c) Determine two (2) set of the Euler circuit of graph shown in Figure 1.

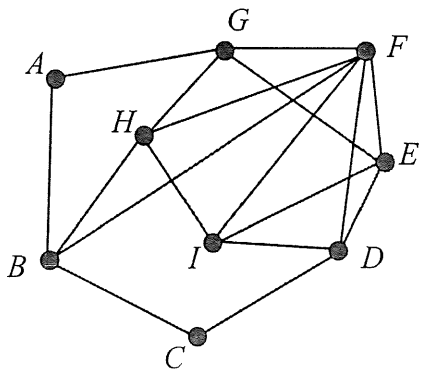

Figure 1 (4 marks)

QUESTION 5

a) Draw a binary tree to represent the infix form of an expression $(a+(b/c))-(d+(e*f))$. (3 marks)

b) Find a minimum-weight spanning tree of the weighted graph in Figure 2 using Kruskal's algorithm.

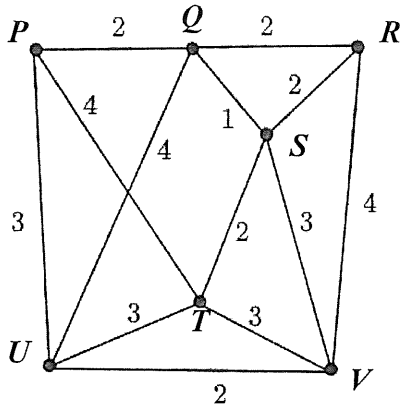

Figure 2

(6 marks)

QUESTION 6

a) Calculate the output value of the following Boolean expression,

$$F(x_1, x_2, x_3) = (\overline{x}_1 + x_3) \cdot \left(\overline{(x_1 \cdot x_2)} + (\overline{x}_2 \cdot x_3) \right)$$

if the Boolean variables are $x_1 = 0$, $x_2 = 1$ and $x_3 = 1$. (2 marks)

b) Draw a circuit of Boolean expression, $f(x, y, z) = \overline{x}(yz + \overline{z})$. Then, construct a logic truth table for the output. (6 marks)

QUESTION 7

A circuit is shown as in Figure 3.

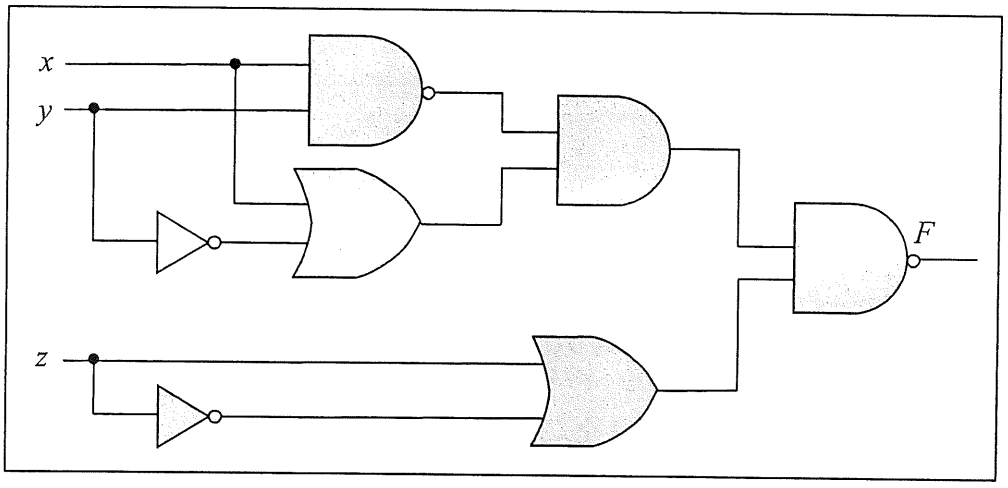

Figure 3

a) Find the expression for the Boolean function in Figure 3. (3 marks)

b) Construct the truth table for the output, F. (4 marks)

c) Simplified the Boolean function and sketch the circuit for the simplified function. (6 marks)

---------- **END OF QUESTIONS** ----------

FORMULA

$a = dq + r$

$q = a$ div d

$r = a$ mod d

Complement:	$\overline{0} = 1$,	$\overline{1} = 0$,	$0 = F$,	$1 = T$
Boolean Sum:	$1+1=1$,	$1+0=1$,	$0+1=1$,	$0+0=0$
Boolean Product:	$1 \cdot 1 = 1$,	$1 \cdot 0 = 0$,	$0 \cdot 1 = 0$,	$0 \cdot 0 = 0$