CONFIDENTIAL

UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	: DGE 1113
COURSE	: MATHEMATICS I
SEMESTER/SESSION	: 1 – 2024/2025
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **7** questions. Answer all questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

QUESTION 1

a) Solve x for the following inequalities:

i) $2-5x<3x-14$ (2 marks)

ii) $-2<\frac{x+1}{3}\leq 4$ (3 marks)

b) Simplify:

i) $\left(4x^2y^3\right)\left(3x^5y\right)$ (2 marks)

ii) $\frac{\left(3x^5y^3z^2\right)^3}{27x^3yz^2}$ (3 marks)

c) Simplify $\left(7-5\sqrt{7}\right)\left(3+5\sqrt{7}\right)$. (3 marks)

d) Given $\log_a 3=m$ and $\log_a 5=n$. Express $\log_a 225$ in terms of m and n.

(3 marks)

e) Solve x for $\log_2(x+1)+\log_2 4=6$. (5 marks)

QUESTION 2

Given $z_1=2+6i$ and $z_2=4+i$ are the complex numbers.

a) Find the complex number z for $\frac{z_1}{z_2}$ and give the answer in standard form, $a+bi$. (3 marks)

b) Calculate the modulus and argument of z. (3 marks)

c) Express the complex number z in polar form. (2 marks)

QUESTION 3

a) By using quadratic formula, solve $9x^2 + 6 = 20x$. (3 marks)

b) Divide $9x^3 - 15x^2 + 4x + 2$ by $3x + 2$ using long division. (4 marks)

c) Express $\dfrac{4x^2 + 7x + 5}{(x+2)(x+3)^2}$ as partial fractions. (7 marks)

QUESTION 4

a) Given $A = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 6 & 3 \\ -2 & 1 \end{bmatrix}$. Find $4B + AC$. (4 marks)

b) Find the values of x, y, u and v for the matrix equation of:

$$\begin{bmatrix} 5x + y & 2u - 3v \\ 4x - 2y & u - 2v \end{bmatrix} = \begin{bmatrix} 18 & 2 \\ 6 & 4 \end{bmatrix}$$

(4 marks)

c) Solve the following simultaneous equations by using the matrix method:

$$5x + 7y = 3$$
$$2x + 3y = -2$$

(6 marks)

QUESTION 5

a) Given a right triangle ABC and $\cos\theta = \dfrac{4}{15}$. Find $\sin\theta$ and $\tan\theta$. (3 marks)

b) Solve $2\cos\theta + 1 = 1.2814$ in the interval of $0° \le \theta \le 360°$. (5 marks)

c) Given a triangle ABC with $a = 6.5cm$, $b = 9cm$ and $c = 14cm$.

   i) Find $\measuredangle A$, $\measuredangle B$ and $\measuredangle C$ by using cosine rule. (10 marks)

   ii) Calculate the triangle area. (3 marks)

QUESTION 6

a) Let $\vec{v} = (2,-4,5)$ and $\vec{w} = (4,3,-2)$. Find $3\vec{v} - 7\vec{w}$. (3 marks)

b) Given the vectors $\vec{a} = 3i + 2j - 2k$ and $\vec{b} = -2i + 5j + 3k$. Calculate the angle, θ between $\vec{a}$ and $\vec{b}$. (5 marks)

QUESTION 7

a) A circle has an area of $550cm^2$. Calculate the radius and circumference of this circle. (4 marks)

b) A closed cylindrical water tank has a radius of 3m and a height of 15m.

   i) Find the surface area of the cylindrical tank. (2 marks)

   ii) Find the volume of the cylindrical tank. (2 marks)

   iii) If the tank is filled with water to a height of 8.5m, calculate the litres of water in the tank. (2 marks)

c) A basketball with a diameter of 24cm needs to be pumped with air into it.

   i) How many cm^3 of air are needed to be pumped in? (2 marks)

   ii) Find the surface area of the basketball. (2 marks)

--------- END OF QUESTIONS --------

FORMULA

$x^m . x^n = x^{m+n}$	$x^m \div x^n = x^{m-n}$
$\left(x^m\right)^n = x^{mn}$	$(xy)^n = x^n y^n$
$x^{\frac{1}{n}} = \sqrt[n]{x}$	$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$
$x^{-n} = \dfrac{1}{x^n}$	$x^{\frac{m}{n}} = \sqrt[n]{x^m}$
$\sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$	$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$
$a\sqrt{b} \pm c\sqrt{b} = (a \pm c)\sqrt{b}$	$\left(\sqrt{a}+\sqrt{b}\right)^2 = a+b+2\sqrt{ab}$
$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = a-b$	$\log_a xy = \log_a x + \log_a y$
$\log_a \dfrac{x}{y} = \log_a x - \log_a y$	$\log_a x^n = n \log_a x$
$\log_a a = 1$	$a^{\log_a x} = x$
$\log_a \left(\dfrac{1}{x}\right) = -\log_a x$	$\log_a m = \log_a n \Leftrightarrow m = n$
$\log_a c = \dfrac{\log_b c}{\log_b a}$	$\log_a b = \dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a}$
$\dfrac{z_1}{z_2} = \dfrac{a+bi}{c+di} \bullet \dfrac{c-di}{c-di}$	$z = r(\cos\theta + i\sin\theta)$ $r = \lvert z\rvert = \sqrt{a^2+b^2}$
$\theta = Arg(z) = \tan^{-1}\left(\dfrac{b}{a}\right)$ $\theta = Arg(z) = \pi - \tan^{-1}\left(\dfrac{b}{a}\right)$ $\theta = Arg(z) = -\left(\pi - \tan^{-1}\left(\dfrac{b}{a}\right)\right)$ $\theta = Arg(z) = -\tan^{-1}\left(\dfrac{b}{a}\right)$	$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\lvert A\rvert = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$	$A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
$X = A^{-1}B$	$x = \dfrac{\lvert A_x\rvert}{\lvert A\rvert}, y = \dfrac{\lvert A_y\rvert}{\lvert A\rvert}$
$H^2 = O^2 + A^2$ $\sin\theta = \dfrac{O}{H}$ $\cos\theta = \dfrac{A}{H}$ $\tan\theta = \dfrac{O}{A}$	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
$Area = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}bc\sin A = \dfrac{1}{2}ac\sin B$	$a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$
$\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$ $\vec{u} - \vec{v} = (u_1 - v_1, u_2 - v_2, u_3 - v_3)$ $\vec{u} \bullet \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$	$\lvert\vec{u}\rvert = \sqrt{u_1^{\,2} + u_2^{\,2} + u_3^{\,2}}$
$\theta = \cos^{-1}\left(\dfrac{\vec{u}\bullet\vec{v}}{\lvert\vec{u}\rvert\lvert\vec{v}\rvert}\right)$	*Circumference circle,* $C = 2\pi r$ *Area circle,* $A = \pi r^2$
Area cylinder: $A = 2\pi rh + 2\pi r^2$ $A = 2\pi rh + \pi r^2$ $A = 2\pi rh$ *Volume cylinder,* $V = \pi r^2 h$	*Area sphere,* $A = 4\pi r^2$ *Volume sphere,* $V = \dfrac{4}{3}\pi r^3$